Phonon-assisted resonant tunneling through a triple-quantum-dot: a phonon-signal detector

X. Y. Shen, Bing Dong, X. L. Lei
Department of Physics, Shanghai Jiaotong University, 1954 Huashan Road, Shanghai 200030, China

PACS. 73.63.Kv – Quantum dots.
PACS. 71.38.-k – Polarons and electron-phonon interaction.
PACS. 73.50.Td – Noise processes and phenomena.

Abstract. – We study the effect of electron-phonon interaction on current and zero-frequency shot noise in resonant tunneling through a series triple-quantum-dot coupling to a local phonon mode by means of a nonperturbative mapping technique along with the Green function formulation. By fixing the energy difference between the first two quantum dots to be equal to phonon frequency and sweeping the level of the third quantum dot, we find a largely enhanced current spectrum due to phonon effect, and in particular we predict current peaks corresponding to phonon-absorption and -emission assisted resonant tunneling processes, which shows that this system can be acted as a sensitive phonon-signal detector or as a cascade phonon generator.

Phonon-assisted inelastic tunneling in semiconductor quantum dot (QD) system at low temperature has become a focus issue in recent years. [1–10] In particular, a recent experiment has measured the nonlinear tunneling through a double-QD (DQD) with the observation of spontaneous phonon emission leading to an additional satellite peak in the current spectrum, [1] which can be ascribed to an interference effect of the electron-phonon interaction (EPI) in a DQD via nonperturbative theoretical analyses. [2–4] This experiment opens a possibility of designing DQD as a coherent phonon generator. However, the phonon-assisted peak in current spectrum is quite fragile and thus detection of phonon-signal is a difficult task in a DQD. [1]

In this letter, we propose a setup containing a triple QD in series coupled to a common local phonon bath and two normal leads, in which energy difference between the first two QDs is fixed to be equal to the phonon frequency, i.e., head of the device acts as a phonon emitter when there is a nonequilibrium current flowing through as suggested by Fujisawa et al. [1] Intuitively, it is imaginable that if the energy of the third QD is tuned, via applying gate voltage, to be higher than the second QD with one-phonon-energy (case a in Fig. 1 below), the emitted phonon could be re-absorbed by electron to help electron tunneling through QD 3 resonantly, resulting in a phonon-absorption-assisted enhanced peak in current spectrum. That is to say that QD 3 *detects* the generated phonon. On the other hand, we predict a significant enhancement of current provided that the energy of QD 3 is further lower than QD 2 by one-phonon-energy (case c in Fig. 1), showing that *more* phonon quanta are generated in tunneling process.

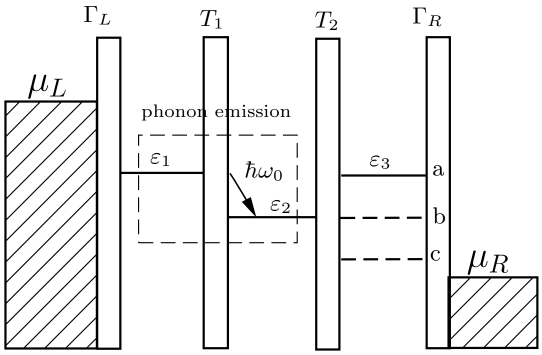

Fig. 1 – Schematic description of tunneling of a triple-QD in the presence of electron-phonon interaction. $\epsilon_1 - \epsilon_2 = \hbar\omega_0$; a, b, and c denotes $\epsilon_3 - \epsilon_2 = \hbar\omega_0$, 0, and $-\hbar\omega_0$, respectively. $\mu_L(\mu_R)$ responds to chemical potentials of the lead L(lead R). Γ_L and Γ_R stands for the coupling between the lead with QD. T_1 and T_2 is the coupling between QDs.

Our setup is schematically shown in Fig. 1. We consider a triple QD (consisting of QD 1, QD 2, and QD 3) which are connected via a tunnel barrier. QD 1 and QD 3 are connected to an electron reservoir in thermal equilibrium with chemical potentials μ_L(source) and μ_R(drain), respectively, with $\mu_L > \mu_R$. Its Hamiltonian can be written as

$$H = H_L + H_R + H_{cen} + H_T, \tag{1}$$

where $H_L + H_R$ describe the two leads, H_{cen} represents the central region consisting of a triple QD in series, and H_T is the coupling of the dots to the lead, respectively:

$$H_L + H_R = \sum_{\eta \in L,R;k} \epsilon_{\eta k} c^\dagger_{\eta k} c_{\eta k}, \tag{2}$$

$$H_{cen} = \sum_\alpha \epsilon_\alpha d^\dagger_\alpha d_\alpha + \hbar\omega_0 b^\dagger b - \sum_\alpha \lambda_\alpha d^\dagger_\alpha d_\alpha (b^\dagger + b) - T_1(d^\dagger_1 d_2 + d^\dagger_2 d_1) - T_2(d^\dagger_2 d_3 + d^\dagger_3 d_2), \tag{3}$$

$$H_T = \sum_k (V_L c^\dagger_{Lk} d_1 + V_R d^\dagger_3 c_{Rk} + \text{H.c.}), \tag{4}$$

where $c^\dagger_{\eta k}(c_{\eta k})$ $(\eta = \{L, R\})$ and $d^\dagger_\alpha(d_\alpha)$ denote creation (annihilation) operators for spinless electrons with momentum k and energy $\epsilon_{\eta k}$ in the left and right leads, and for spinless electrons on the αth $(\alpha = \{1, 2, 3\})$ QD, respectively. ϵ_α is the energy level of the αth QD, and $T_1(T_2)$ stands for the interdot hopping between the QD 1 and QD 2 (QD 2 and QD 3). $V_{L(R)}$ is the coupling constant of the central regime with lead L(R). This sort of triple-QD system was already realized in experiments about ten years ago, in which the energy levels of every QD and dot-dot hoppings can all be adjusted by applying gate voltages. [11] The operator $b^\dagger(b)$ in Eq. (3) creates (destroys) a phonon, and λ_α is the interaction constant of the electron on the αth dot with phonon. Here, we consider the phonon bath as a single-mode phonon with dispersionless energy $\hbar\omega_0$, and assume that phonon remains coherence during electron tunneling processes, i.e., we take no account of phonon dissipation in the present investigation. For simplicity, we also ignore electron-electron interaction in this letter.

As the system under study contains a many-body problem of electron-phonon scattering during the single-electron tunneling process, it can not be solved analytically. Applying the

nonperturbative mapping technique suggested by Bonča and Trugman [5], we can transform the many-body problem into a multichannel one-body problem by transforming the Hamiltonian Eq. (1) in terms of a new set of electron states combined of single-electron states and n-phonon states. We define a direct product states with single-electron states and n-phonon Fock states as:

$$|\alpha, n\rangle = d_\alpha^\dagger \frac{(b^\dagger)^n}{\sqrt{n!}}|0\rangle, \tag{5}$$

and

$$|\eta k, n\rangle = c_{\eta k}^\dagger \frac{(b^\dagger)^n}{\sqrt{n!}}|0\rangle, \tag{6}$$

such that the electron state $|\alpha\rangle$ in QD and the state of electron in leads $|\eta k\rangle$ are accompanied by n phonons ($|0\rangle$ is the vacuum state). We perform this transformation on the electron-phonon interaction part of Eq. (3), so the many body problem can be mapped onto a one-body model:

$$\sum_{\alpha n} -\lambda_\alpha \sqrt{n+1}(|\alpha, n+1\rangle\langle\alpha, n| + |\alpha, n\rangle\langle\alpha, n+1|). \tag{7}$$

With defining these Dirac bracket as new operators:

$$d_{\alpha n}^\dagger = |\alpha, n\rangle, c_{\eta k n}^\dagger = |\eta k, n\rangle, \tag{8}$$

we obtain:

$$\sum_{\alpha n} \lambda_\alpha \sqrt{n+1}(d_{\alpha n+1}^\dagger d_{\alpha n} + d_{\alpha n}^\dagger d_{\alpha n+1}). \tag{9}$$

We can do the same transformation on the Hamiltonian Eq. 1 as $\widetilde{H} = \widetilde{H}_L + \widetilde{H}_R + \widetilde{H}_{cen} + \widetilde{H}_T$: [4]

$$\widetilde{H}_L + \tilde{H}_R = \sum_{\eta \in L,R;kn} \epsilon_{\eta k n} c_{\eta k n}^\dagger c_{\eta k n}, \tag{10}$$

$$\widetilde{H}_{cen} = \sum_{\alpha n} \epsilon_{\alpha n} d_{\alpha n}^\dagger d_{\alpha n} - \sum_{\alpha n} \lambda_\alpha \sqrt{n+1}(d_{\alpha n+1}^\dagger d_{\alpha n} + \text{H.c.}) - T_1(d_{1n}^\dagger d_{2n} + \text{H.c.}) - T_2(d_{2n}^\dagger d_{3n} + \text{H.c.}), \tag{11}$$

$$\widetilde{H}_T = \sum_{kn}(V_{Ln} c_{Lkn}^\dagger d_{1n} + V_{Rn} d_{3n}^\dagger c_{Rkn} + \text{H.c.}), \tag{12}$$

where $d_{\alpha n}^\dagger$ and $c_{\eta k n}^\dagger$ are new operators with phonon quanta n, $\epsilon_{\alpha n} = \epsilon_\alpha + n\hbar\omega_0$, $\epsilon_{\eta k n} = \epsilon_{\eta k} + n\hbar\omega_0$. It is important to note that the channel indices stand for the phonon quanta, so we have to add a weight factor $P_n = (1 - e^{-\hbar\omega_0/k_BT})e^{-n\hbar\omega_0/k_BT}$ to the nth channel.

We define the retarded GFs of the triple QDs, $G^r_{\alpha\beta,mn}(t,t') = -i\theta(t-t')\langle\{d_{\alpha m}(t), d_{\beta n}^\dagger(t')\}\rangle$ $(\alpha, \beta = 1, 2, 3)$, $G^r_{\eta k\alpha,mn}(t,t') = -i\theta(t-t')\langle\{c_{\eta k m}(t), d_{\alpha n}^\dagger(t')\}\rangle$ where m, n represents the phonon quanta. Employing the equation-of-motion technique, we get the retarded GFs in the matrix form:

$$\begin{pmatrix} \boldsymbol{G}_{11}^r & \boldsymbol{G}_{12}^r & \boldsymbol{G}_{13}^r \\ \boldsymbol{G}_{21}^r & \boldsymbol{G}_{22}^r & \boldsymbol{G}_{23}^r \\ \boldsymbol{G}_{31}^r & \boldsymbol{G}_{32}^r & \boldsymbol{G}_{33}^r \end{pmatrix} = \begin{pmatrix} (\omega + \frac{i}{2}\Gamma_L)\boldsymbol{I} - \boldsymbol{A}_1 & T_1\boldsymbol{I} & 0 \\ T_1\boldsymbol{I} & \omega\boldsymbol{I} - \boldsymbol{A}_2 & T_2\boldsymbol{I} \\ 0 & T_2\boldsymbol{I} & (\omega + \frac{i}{2}\Gamma_R)\boldsymbol{I} - \boldsymbol{A}_3 \end{pmatrix}^{-1}, \tag{13}$$

in which ω responds to energy, and $\boldsymbol{I}$ is a $N \times N$ unit matrix and $\boldsymbol{A}_\alpha$ is a $N \times N$ symmetrical tridiagonal matrix with $\boldsymbol{A}_{\alpha;nn} = \epsilon_\alpha + n\hbar\omega_0$, $\boldsymbol{A}_{\alpha;n(n-1)} = -\lambda_\alpha\sqrt{n}$, and $\boldsymbol{A}_{\alpha;n(n+1)} = -\lambda_\alpha\sqrt{n+1}$,

respectively. $\Gamma_\eta = 2\pi \sum_k |V_\eta|^2 \delta(\omega - \epsilon_{\eta k} - m\hbar\omega_0)$ with the wide band limit represents the coupling strength of the center region with lead η.

The current of the nth channel in the left lead through the center region can be obtained from the time evolution of the occupation number operator of the left lead: $I = -e\langle \dot{N}_L \rangle = -\frac{ie}{\hbar}\langle [\tilde{H}, N_L] \rangle$ with $N_L = \sum_{k,n} c^\dagger_{Lkn} c_{Lkn}$. After some algebra, we find $I = \frac{ie}{\hbar} \sum_{k,n} [V_{Ln} \langle c^\dagger_{Lkn} d_{1n} \rangle - V^*_{Ln} \langle d^+_{1n} c_{Lkn} \rangle]$. Using the Keldysh nonequilibrium Green's function (GF) technique and considering that the total current is a sum over all pseudo-channels accompanied with the weight factor P_n, we get the current as: [4–6]

$$I = \frac{e}{h} \int d\omega \sum_{mn} (t^+_{Lmn} t_{Lmn}) \{ P_n f^n_L(\omega)[1 - f^m_R(\omega)] - P_m f^m_R(\omega)[1 - f^n_L(\omega)] \}, \tag{14}$$

where $f_{L(R)}(\omega) = (1 + e^{(\omega - \mu_{L(R)})/k_B T})^{-1}$ is the Fermi distribution of the leads at local thermal equilibrium, inwhich T is the temperature. t_{Lnm} represents the transmissionprobability of a electron through the center region from the nth channel of the left lead to the mth channel of the right lead. According to the Fish-Lee relation relating the scatting matrix elements with the retarded GFs, [12] we describe the transmission and reflection probabilities in terms of retarded GFs Eq. (13):

$$\begin{aligned} r_{L(R),mn} &= -\delta_{mn} + i\Gamma G^r_{11(33),mn}(\omega), && (15) \\ t_{L(R),mn} &= i\Gamma G^r_{31(13),mn}(\omega). && (16) \end{aligned}$$

Furthermore, we use the Büttiker scatting method to calculate the shot noise [13] and follow our previous work, [4] we obtain the zero-frequency shot noise of the system:

$$\begin{aligned} S_{LL}(0) &= \frac{2e^2}{h} \int d\omega \sum_{mn} \{ |(t^\dagger_L t_L)_{mn}|^2 P_n f^n_L(\omega)(1 - f^m_L(\omega)) + |(t^\dagger_L r_L)_{mn}|^2 [P_n f^n_L(\omega) \\ &\times (1 - f^m_R(\omega)) + P_m f^m_R(\omega)(1 - f^n_L(\omega))] + |(t^\dagger_R t_R)_{mn}|^2 P_n f^n_R(\omega)(1 - f^m_R(\omega)) \}. \quad (17) \end{aligned}$$

In order to obtain a more physical view of the results, we now make a numerical simulation of the current and the shot noise based on Eqs. (14) and (17) as functions of the energy difference between QD 2 and QD 3: $\epsilon = \epsilon_2 - \epsilon_3$ (which can be tuned by applying gate voltage). With considering the experiment by Fujisawa, [1] we choose the temperature $T = 23$mk ($k_B T = 2\mu$eV), and dispersionless phonon $\hbar\omega_0 = 20\mu$eV. The scale of the dot is $d = c_s/\omega_0 = 175nm$ where $c_s = 5300m/s$ is the longitudinal sound velocity. At low temperature the phonons are assumed to be piezoelectric acoustical mode with interaction constant $|\lambda|^2 = \frac{1}{2\pi} \frac{\hbar P}{2\rho V \omega_0}$, where P is the piezoelectric coupling, ρ is the ion mass density, V is the volume. [14] Employing the typical GaAs parameters, [14] we get $\lambda \sim 1.6\mu$eV. We find the electron-phonon interaction constant $\lambda_1, \lambda_2, \lambda_3$ on the three quantum dots coincide up to a phase factor. [2] For simplicity, we choose phase factor is -1 between the QD 1 and QD 2 (QD 2 and QD 3), so we get $\lambda_1 = -\lambda_2 = \lambda_3 = 1.6\mu$eV (This can be done by tuning size of each dot and distance between them in experiments). In this cases, we set the couplings of the dots with the two leads to be symmetric, $\Gamma_L = \Gamma_R = \Gamma = 4\mu$eV, and the interdot hoppings to be equal, $T_1 = T_2 = T_c = 4\mu$eV. We assume $\epsilon_2 = 0\mu$eV, which is equal to the Fermi energies of the left lead and right lead at equilibrium. We also set the energy gap of the QD 1 and QD 2 as $\epsilon_1 - \epsilon_2 = \hbar\omega_0 = 20\mu$eV and the symmetrically applied bias voltage $\mu_L - \mu_R = eV_{sd} = 200\mu$eV so that $\mu_L \gg \epsilon_1, \epsilon_2, \epsilon_3 \gg \mu_R$, in which condition sufficient more pseudo-channels are involved in transport and thus our numerical results are nearly independent of the bias-voltage.

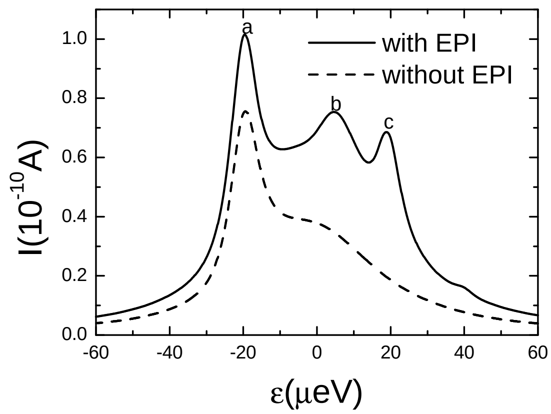

Fig. 2 – Calculated total tunneling current versus energy difference ϵ with parameter: $\Gamma = 4\mu$eV and $\lambda_1 = -\lambda_2 = \lambda_3 = 1.6\mu$eV, the temperature of the system $T = 23$mK.

In Fig.2 we plot the total current of the triple-QD system, for comparison, in the absence of EPI (dash line) vs the energy difference ϵ. It is interesting to find a peak at $\epsilon = \epsilon_2 - \epsilon_3 = -\hbar\omega_0$ (where $\epsilon_1 = \epsilon_3$) in the system without considering electron-phonon interaction. This peak is due to a resonant tunneling between QD1 and QD3 via QD 2. We can describe the dynamics of the triple QD system with quantum rate equations for the density matrix of the dots. [15] With solving the master equation algebraically, we get the current at zero temperature analytically as:

$$I = \frac{e}{\hbar} \frac{4\Gamma T_c^4}{16T_c^4 + (2\Gamma^2 + 8\epsilon^2 + 16\epsilon\epsilon_{12} + 8\epsilon_{12}^2)T_c^2 + 4\epsilon_{12}^4 + \epsilon_{12}^2\Gamma^2 + 4\epsilon_{12}^2\epsilon^2 + 8\epsilon_{12}^3\epsilon}, \quad (18)$$

with $\epsilon_{12} = \epsilon_1 - \epsilon_2$. It is obvious to obtain the peak of current at $\epsilon = -\epsilon_{12} = \hbar\omega_0$.

In Fig.2 we also plot the total current of the triple-QD system in the presence of EPI vs the energy difference ϵ. Similar with the current spectrum of a double-QD system, [1–4] an obvious overall enhancement of current is found for the triple-QD system with phonon bath in comparison with that without EPI. Moreover, as above expected, our results predict three current peaks labeled by a, b, and c, which are corresponding to the three configurations described in Fig. 1. When an extremely large bias-voltage is applied to the whole system, an electron can resonantly jump from QD 1 to QD 2 accompanied by one-phonon generated, then if the system is at configuration b, $\epsilon_2 = \epsilon_3$, the electron can directly resonantly tunnel through the device and the current peak b is caused by phonon emission between QD1 and QD2; however, if the system is at configuration c, $\epsilon_2 - \epsilon_3 = \hbar\omega_0$, the electron will emit another phonon to reach QD 3, leading to two-phonon-emission peak c; more interestingly, if the system is at configuration a, $\epsilon_2 - \epsilon_3 = -\hbar\omega_0$, an enhanced current peak a also occurs stemming from re-absorption of the generated phonon in head of the device by electron to overcome the barrier between QD 2 and QD 3, which is enhanced by 30% in comparison to the resonant peak without EPI. Of course, phonon dissipation to environment could lessen these phonon-assisted resonant peaks. Albeit we do not consider nonequilibrium phonon effect [7, 8] and dissipation due to environment, [9, 10] however, it can be expected from our above calculations that the multi-QD system could function as a cascade phonon generator with higher emission efficiency than a double-QD or as a sensitive and "good" phonon-signal detector. Actually, we can put this system into a phonon resonant cavity in experiments to lessen the leakage of phonon and thus to enhance the probability of re-absorption of phonon.

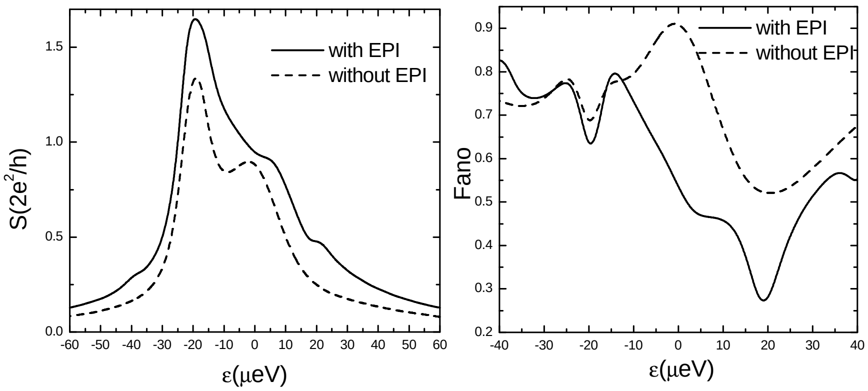

Fig. 3 – (a) Zero-frequency shot noise versus energy difference ϵ with the same parameter of Fig.2; (b) Fano factor versus energy difference ϵ.

These studies will be the subject in our future work.

We also examine the zero-frequency shot-noise (a) and Fano factor $F = S_{LL}/2eI$ (b) with the energy difference ϵ, as plotted in Fig.3. Corresponding to the three current peaks due to phonon-absorption or -emission, the shot noise spectrum shows also single peak in the phonon-assisted resonant tunneling region. As the shot noise $S \sim T_{tr}(1 - T_{tr})$, which T_{tr} is transmission probability, there is no shot noise generation when the $T_{tr} = 1$ or 0. The maximal generation of the shot noise occurs while the transmission probability is between 0 and 1. We also find a small peak at $\epsilon = 0$ in the absence of phonon. However, the shoulder at $\epsilon = 0$ is smeared smoothly by the interaction of phonon. The Fano factor F displays three dips at the three points a, b, and c, implying that the inelastic resonance suppresses the shot noise. It is worth noticing that the phonon-absorption-assisted tunneling induces a more pronounced dip.

In summary, we investigate the resonant tunneling through a triple QD in the presence of EPI at low temperature by means of the nonperturbative mapping technique in combination with the nonequilibrium GF method. By making the first two dots act as a phonon emitter and sweeping the energy level of the third QD, we evaluate the tunneling current and predict that resonant peaks in the current spectrum are not only due to spontaneous phonon-emission assisted tunneling, but also due to phonon-absorption process. We also study zero-frequency shot noise of this sytem.

This work was supported by Projects of the National Science Foundation of China, the Shanghai Municipal Commission of Science and Technology, the Shanghai Pujiang Program, and NCET.

REFERENCES

[1] T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. broer, R. Aguado, S. Tarucha, and Leo P. Kouwenhoven, *Science*, **282** (1998) 932.
[2] T. Brandes, and B. Kramer, *Phys. Rev.Lett*, **83** (1999) 3021.
[3] M. Keil, and H. Schoeller, *Phys. Rev. B*, **66** (2002) 155314.
[4] Bing Dong, H. L. Cui, X. L. Lei, and Norman J. M. Horing, *Phys. Rev. B*, **71** (2005) 045331.

[5] J. BONČA, and S. A. TRUGMAN, *Phys. Rev. Lett.*, **79** (1997) 4874; K. HUALE, and J. BONČA, *Phys. Rev. B*, **59** (1999) 13087.
[6] H. NESS, and A.J. FISHER, *Phys. Rev. Lett.*, **83** (1999) 452; E.G. EMBERLY, and G. KIRCZENOW, *Phys. Rev. B*, **61** (2000) 5740.
[7] A. MITRA, I. ALEINER, and A. J. MILLIS, *Phys. Rev. B*, **69** () 245302.
[8] J. KOCH, and F. VON OPPEN, *Phys. Rev. Lett.*, **94** (2005) 206804.
[9] K. FLENSBERG, *Phys. Rev. B*, **68** (2003) 205323; S. BRAIG, and K. FLENSBERG, *Phys. Rev. B*, **68** (2003) 205324.
[10] M. GALPERIN, A. NITZAN, and M. A. RATNER, *Phys. Rev. B*, **73** (2006) 45314.
[11] F. R. WAUGH, M. J. BERRY, D. J. MAR, and R. M. WESTERVELT, *Phys. Rev. Lett.*, **75** (1995) 905.
[12] D. S. FISHER, and P. A. LEE, *Phys. Rev. B*, **23** (1981) 6851.
[13] M. BÜTTIKER, *Phys. Rev. B*, **46** (1992) 12485.
[14] HENRIK BRUUS, KARSTEN FLENSBERG, and HENRIK SMITH, *Phys. Rev. B*, **48** (1993) 11144
[15] F. RENZONI and T. BRANDES, *Phys. Rev. B*, **64** (2001) 245301